

due January 6<sup>th</sup> (Monday)

5.2/ 49-50, 53-54, 57, 62-63, 67

5.3/ 51, 53-55, 59-61, 64, 68-69

5.4/ 16-17, 51-52, 55-57, 61-62, 69-70, 76-77, 81-82,  
85, 114

5.5/ 21-26, 31, 34-37, 41, 50, 59, 62

5.6/ 1-75

Extra Credit <sup>day 72</sup>

Barrons  
chapter 3  
problems at  
end

## 5.6/ Integration using Substitution

day 72

Moving Beyond the Rules

Every derivative rule is an "integration rule"  
in reverse

Often this isn't much help "

But notice:  $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Lurking bts integration rule:

$$\int \underbrace{f'(g(x)) \cdot g'(x)} dx = f(g(x)) + C$$

5.6/13)

day 72

$$\int \boxed{2x} \underbrace{(x^2+1)^4} \boxed{dx}$$

idea

Let  $g'(x) = 2x$   
 $g(x) = x^2 + 1$   
 in this case

Let  $u = x^2 + 1$

then  $du = \boxed{2x dx}$

or  
 $\frac{du}{dx} = 2x$   
 so  
 $du = 2x dx$

$$\begin{aligned} \int 2x(x^2+1)^4 dx \\ &= \int (\sim)^4 d\sim \\ &= \frac{(\sim)^5}{5} + C \\ &= \frac{(x^2+1)^5}{5} + C \end{aligned}$$

$$\begin{aligned} \int u^4 du &= \frac{u^5}{5} + C \\ &= \frac{(x^2+1)^5}{5} + C \end{aligned}$$

day 72

5.6/17)

$$\int \boxed{2x} \underbrace{(x^2-1)}_{\text{replace as many as needed}} \boxed{dx}$$

Let  $u = x^2 - 1$ 

$$\text{then } du = \boxed{2x dx}$$

replace 1e

$$= \frac{(x^2-1)^{100}}{100} + C$$

$$\int u^{99} du$$

$$= \frac{u^{100}}{100} + C$$



18)

$$\int \boxed{x} e^{\boxed{x^2}} \boxed{dx}$$

day 72

$$\int e^u du = e^u + C$$

Let  $u = x^2$   
 then  $du = 2x dx$   
 $\frac{1}{2} du = \boxed{x dx}$

$$\frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$$

$$= \frac{1}{2} e^{x^2} + C$$

Guide  
 Looking for  
 one of  
 the

(14)

day 72

19)

$$\int \frac{2x^2}{\sqrt{1-4x^3}} dx$$

Let  $u = 1 - 4x^3$

$$\frac{du}{-6} = \frac{-12x^2}{-6} dx$$

$$-\frac{1}{6} du = 2x^2 dx$$

$$= -\frac{1}{3} \sqrt{1-4x^3} + C$$

$$\left(-\frac{1}{6}\right) \int \frac{1}{\sqrt{u}} du$$

$$\int \frac{1}{\sqrt{1-?^2}}$$

or

$$\int \frac{1}{\sqrt{u}}$$

$$\left(-\frac{1}{2}\right)$$

$$= -\frac{1}{6} \int u^{-\frac{1}{2}} du$$

$$= -\frac{1}{6} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= -\frac{2}{6} \sqrt{u} + C$$

day 72

24)  $\int \sin^{10} \theta \cos \theta d\theta$

$$\int u^{10} du$$
$$= \frac{u^{11}}{11} + C$$

Let  $u = \sin \theta$

$$du = \cos \theta d\theta$$

[remember -  
 $\sin^{10} \theta = (\sin \theta)^{10}$ ]

$$\frac{\sin^{11} \theta}{11} + C$$

day 72

un  
numbered

$$\int \frac{\cos x}{1 + \sin^2 x} dx$$

$$\text{let } u = \sin x \\ du = \cos x dx$$

$$\tan^{-1}(\sin x) + C$$

$$\int \frac{1}{1+u^2} du \\ = \tan^{-1}(u) + C$$



day 72

creative

1) Decide on a  
u-substitution  
(u = inside  $f^n$ )

\* Consider  
→ inside  $f^n$  and  
derivative of  
inside  $f^n$   
→ where you  
are headed  
(big 14)

technique

2) Take the derivative of  
both sides  
(differential form)

2½) build integral in "u"

3) if you have a basic rule, take antiderivative  
if not, repeat 1-2 or try different  
u =